

ON THE OHSAWA–TAKEGOSHI EXTENSION THEOREM

BY ZBIGNIEW BŁOCKI

Abstract. Motivated by a recent work by B.-Y. Chen we prove a new estimate for the $\bar{\partial}$ -operator, which easily implies the Ohsawa–Takegoshi extension theorem. We essentially only use the classical Hörmander estimate. This method gives the same constant as the one recently obtained by Guan–Zhou–Zhu.

1. Introduction. The Ohsawa–Takegoshi extension theorem [14] turned out to be one of the most important results in complex analysis and geometry. There have been various simplifications of its proof (see e.g. [1]) but the crucial one is due to B.-Y. Chen [8], who recently showed that it follows directly from Hörmander’s estimate for the $\bar{\partial}$ -equation. Using some of his ideas we obtain a generalization of an estimate due to Berndtsson [1] (see Theorem 1 below), from which the Ohsawa–Takegoshi theorem can be deduced directly.

We are also interested in the conjecture formulated by Suita [15]: for a bounded domain D in \mathbb{C} one has

$$c_D^2 \leq \pi K_D.$$

Here

$$c_D(z) = \exp(\lim_{\zeta \rightarrow z} (G_D(\zeta, z) - \log |\zeta - z|)),$$

where $G_D(\cdot, z)$ is the (negative) Green function with pole at $z \in D$, and

$$K_D(z) = \sup\{|f(z)|^2 : f \text{ holomorphic in } D, \int_D |f|^2 d\lambda \leq 1\}$$

is the Bergman kernel. Its relation to the extension theorem was found by Ohsawa [13] who, using methods of the $\bar{\partial}$ -equation, showed the estimate

$$(1) \quad c_D^2 \leq C \pi K_D$$

2010 *Mathematics Subject Classification.* 32A25, 32W05.

Key words and phrases. Bergman kernel, $\bar{\partial}$ -equation.

with $C = 750$. It was improved in [6] to $C = 2$ and recently to $C = 1.95388\dots$ by Guan–Zhou–Zhu, as announced in [11]. We show, see Theorem 4 below, a result which covers both the Ohsawa–Takegoshi result and (1) with the same constant as in [11] (Theorem 4 below was originally shown in [9] with $C = 4$).

2. The estimate for the $\bar{\partial}$ -equation. Let Ω be a pseudoconvex domain in \mathbb{C}^n and assume that

$$\alpha = \sum_j \alpha_j d\bar{z}_j \in L_{loc,(0,1)}^2(\Omega)$$

satisfies $\bar{\partial}\alpha = 0$. We are looking for solutions of

$$(2) \quad \bar{\partial}u = \alpha$$

with L^2 -estimates. The classical one is due to Hörmander [12]: for any plurisubharmonic φ in Ω we can find u with

$$(3) \quad \int_{\Omega} |u|^2 e^{-\varphi} d\lambda \leq \int_{\Omega} |\alpha|_{i\bar{\partial}\bar{\partial}\varphi}^2 e^{-\varphi} d\lambda.$$

For C^2 , strongly plurisubharmonic φ we have

$$|\alpha|_{i\bar{\partial}\bar{\partial}\varphi}^2 = \sum_{j,k} \varphi^{j\bar{k}} \bar{\alpha}_j \alpha_k,$$

where $(\varphi^{j\bar{k}})$ is the inverse transposed of $(\partial^2\varphi/\partial z_j\partial\bar{z}_k)$, whereas for φ which is only plurisubharmonic the right-hand side of (3) is a bit ambiguous. It makes sense however (and the estimate indeed holds – see [4] or [5]) if instead of $|\alpha|_{i\bar{\partial}\bar{\partial}\varphi}^2$ we take any $h \in L_{loc}^\infty(\Omega)$ with

$$i\bar{\alpha} \wedge \alpha \leq h i\bar{\partial}\bar{\partial}\varphi.$$

Berndtsson [1] showed another estimate for (2): if in addition ψ is a plurisubharmonic function in Ω satisfying

$$(4) \quad i\partial\psi \wedge \bar{\partial}\psi \leq i\bar{\partial}\bar{\partial}\psi$$

and $0 < \delta < 1$, then we can find u with

$$(5) \quad \int_{\Omega} |u|^2 e^{\delta\psi - \varphi} d\lambda \leq \frac{4}{\delta(1-\delta)^2} \int_{\Omega} |\alpha|_{i\bar{\partial}\bar{\partial}\psi}^2 e^{\delta\psi - \varphi} d\lambda.$$

The constant in (5) was improved in [3]: it was shown that the optimal $C(\delta)$ satisfies

$$\frac{4}{(1-\delta)(2-\delta)} \leq C(\delta) \leq \frac{4}{(1-\delta)^2}.$$

Then (5) makes sense also for $\delta = 0$: one obtains the following estimate due to Donnelly and Fefferman [10]:

$$\int_{\Omega} |u|^2 e^{-\varphi} d\lambda \leq 4 \int_{\Omega} |\alpha|_{i\bar{\partial}\bar{\partial}\psi}^2 e^{-\varphi} d\lambda.$$

It is also clear that we cannot have a finite constant in (5) $\delta = 1$.

Note that in our convention (4) can be written as $|\bar{\partial}\psi|_{i\bar{\partial}\bar{\partial}\psi}^2 \leq 1$. Keeping this in mind we will formulate our main result which can be viewed as a variant of Berndtsson's estimate (5) for $\delta = 1$:

THEOREM 1. *Assume that Ω is a pseudoconvex domain in \mathbb{C}^n and take $\alpha \in L_{loc,(0,1)}^2(\Omega)$ with $\bar{\partial}\alpha = 0$. Let φ, ψ be plurisubharmonic functions in Ω such that $|\bar{\partial}\psi|_{i\bar{\partial}\bar{\partial}\psi}^2 \leq 1$ in Ω and $|\bar{\partial}\psi|_{i\bar{\partial}\bar{\partial}\psi}^2 \leq \delta < 1$ on $\text{supp } \alpha$. Then there exists $u \in L_{loc}^2(\Omega)$ solving $\bar{\partial}u = \alpha$ and such that*

$$(6) \quad \int_{\Omega} (1 - |\bar{\partial}\psi|_{i\bar{\partial}\bar{\partial}\psi}^2) |u|^2 e^{\psi-\varphi} d\lambda \leq \frac{1}{(1 - \sqrt{\delta})^2} \int_{\Omega} |\alpha|_{i\bar{\partial}\bar{\partial}\psi}^2 e^{\psi-\varphi} d\lambda.$$

PROOF. By standard approximation we may assume that φ, ψ are smooth up to the boundary. We now use a trick from [2]. Let u be the minimal solution to $\bar{\partial}u = \alpha$ in $L^2(\Omega, e^{-\varphi})$. This is equivalent to u being perpendicular to $\ker \bar{\partial}$ in $L^2(\Omega, e^{-\varphi})$. Therefore, $v := ue^{\psi}$ is perpendicular to $\ker \bar{\partial}$ in $L^2(\Omega, e^{-\varphi-\psi})$, which means that v is the minimal solution to $\bar{\partial}v = \beta$, where

$$\beta := (\alpha + u\bar{\partial}\psi)e^{\psi},$$

in $L^2(\Omega, e^{-\varphi-\psi})$. Therefore, by Hörmander's estimate (3)

$$\begin{aligned} \int_{\Omega} |u|^2 e^{\psi-\varphi} d\lambda &= \int_{\Omega} |v|^2 e^{-\varphi-\psi} d\lambda \\ &\leq \int_{\Omega} |\beta|_{i\bar{\partial}\bar{\partial}(\varphi+\psi)}^2 e^{-\varphi-\psi} d\lambda \leq \int_{\Omega} |\alpha + u\bar{\partial}\psi|_{i\bar{\partial}\bar{\partial}\psi}^2 e^{\psi-\varphi} d\lambda. \end{aligned}$$

Denoting $h := |\bar{\partial}\psi|_{i\bar{\partial}\bar{\partial}\psi}^2$, for any $t > 0$ we get

$$\begin{aligned} &\int_{\Omega} |\alpha + u\bar{\partial}\psi|_{i\bar{\partial}\bar{\partial}\psi}^2 e^{\psi-\varphi} d\lambda \\ &\leq (1 + t^{-1}) \int_{\Omega} |\alpha|_{i\bar{\partial}\bar{\partial}\psi}^2 e^{\psi-\varphi} d\lambda + t \int_{\text{supp } \alpha} |u|^2 h e^{\psi-\varphi} d\lambda + \int_{\Omega} |u|^2 h e^{\psi-\varphi} d\lambda \\ &\leq (1 + t^{-1}) \int_{\Omega} |\alpha|_{i\bar{\partial}\bar{\partial}\psi}^2 e^{\psi-\varphi} d\lambda + \delta(t + 1) \int_{\text{supp } \alpha} |u|^2 e^{\psi-\varphi} d\lambda \\ &\quad + \int_{\Omega \setminus \text{supp } \alpha} |u|^2 h e^{\psi-\varphi} d\lambda. \end{aligned}$$

Therefore

$$\begin{aligned} &\int_{\Omega \setminus \text{supp } \alpha} (1 - h) |u|^2 e^{\psi-\varphi} d\lambda + (1 - \delta(t + 1)) \int_{\text{supp } \alpha} |u|^2 e^{\psi-\varphi} d\lambda \\ &\leq (1 + t^{-1}) \int_{\Omega} |\alpha|_{i\bar{\partial}\bar{\partial}\psi}^2 e^{\psi-\varphi} d\lambda. \end{aligned}$$

Since the left-hand side is bounded below by

$$(1 - \delta(t + 1)) \int_{\Omega} (1 - h) |u|^2 e^{\psi - \varphi} d\lambda$$

for $t = \delta^{-1/2} - 1$ we get (6). \square

Note that, after replacing ψ by $\delta\psi$, Theorem 1 gives the Berndtsson estimate (5) with the constant

$$\frac{1}{\delta(1 - \delta)(1 - \sqrt{\delta})^2}.$$

3. The Ohsawa–Takegoshi extension theorem. The following lemma is essentially contained in [8]:

LEMMA 2. For $\zeta \in \mathbb{C}$ with $|\zeta| \leq (2e)^{-1/2}$ and $\varepsilon > 0$ sufficiently small, set

$$\psi(\zeta) := -\log \left[-\log(|\zeta|^2 + \varepsilon^2) + \log \left(-\log(|\zeta|^2 + \varepsilon^2) \right) \right].$$

Then ψ is subharmonic in $\{|\zeta| < (2e)^{-1/2}\}$ and there exist constants C_1, C_2, C_3 such that

- i) $(1 - \frac{|\psi_{\zeta}|^2}{\psi_{\zeta\bar{\zeta}}}) e^{\psi} \geq \frac{1}{C_1 \log^2(|\zeta|^2 + \varepsilon^2)}$ on $\{|\zeta| \leq (2e)^{-1/2}\}$;
- ii) $\frac{|\psi_{\zeta}|^2}{\psi_{\zeta\bar{\zeta}}} \leq \frac{C_2}{-\log \varepsilon}$ on $\{|\zeta| \leq \varepsilon\}$;
- iii) $\frac{e^{\psi}}{|\zeta|^2 \psi_{\zeta\bar{\zeta}}} \leq C_3$ on $\{\varepsilon/2 \leq |\zeta| \leq \varepsilon\}$.

Using Theorem 1 and Lemma 2 similarly as in [8] we can easily prove an extended version of the Ohsawa–Takegoshi theorem:

THEOREM 3. Assume that $\Omega \subset \mathbb{C}^{n-1} \times \{|z_n| < (2e)^{-1/2}\}$ is pseudoconvex and let φ be a plurisubharmonic function in Ω . Then every holomorphic f in $\Omega' := \Omega \cap \{z_n = 0\}$ (we assume that Ω' is not empty) has a holomorphic extension F in Ω satisfying

$$\int_{\Omega} \frac{|F|^2 e^{-\varphi}}{|z_n|^2 \log^2 |z_n|^2} d\lambda \leq C \int_{\Omega'} |f|^2 e^{-\varphi} d\lambda',$$

where C is a uniform constant.

PROOF. We follow the argument from [8]. By standard approximation we may assume that Ω is bounded with smooth boundary, φ is smooth up to the boundary and f is defined in a neighborhood of $\bar{\Omega}'$. Let $\chi \in C^\infty(\mathbb{R})$ be such that $\chi = 1$ on $\{t \leq 1/2\}$ and $\chi = 0$ on $\{t \geq 1\}$. For small $\varepsilon > 0$ set

$$\alpha := \bar{\partial}(f(z')\chi(|z_n|^2/\varepsilon^2)) = f(z')\chi'(|z_n|^2/\varepsilon^2)z_n d\bar{z}_n/\varepsilon^2.$$

We use Theorem 1 with ψ given by Lemma 2 and $\tilde{\varphi} = \varphi + 2 \log |z_n|$. With $\delta := -C_2/\log \varepsilon$, we get a solution $u = u_\varepsilon$ to (2) with

$$\int_{\Omega} \frac{|u|^2 e^{-\varphi}}{|z_n|^2 \log^2(|z_n|^2 + \varepsilon^2)} d\lambda \leq \frac{C_1}{(1 - \sqrt{\delta})^2} \int_{\Omega} |\alpha|_{i\partial\bar{\partial}\psi}^2 e^{\psi - \tilde{\varphi}} d\lambda.$$

We have

$$\int_{\Omega} |\alpha|_{i\partial\bar{\partial}\psi}^2 e^{\psi - \tilde{\varphi}} d\lambda \leq \frac{(\sup \chi')^2}{\varepsilon^4} \int_{\{\frac{\varepsilon}{2} \leq |z_n| \leq \varepsilon\}} \frac{e^{\psi}}{\psi_{\zeta\bar{\zeta}}} d\lambda \sup_{\frac{\varepsilon}{2} \leq |\zeta| \leq \varepsilon} \int_{\Omega'_\zeta} |f|^2 e^{-\varphi} d\lambda',$$

where $\Omega'_\zeta = \{z' \in \mathbb{C}^{n-1} : (z', \zeta) \in \Omega\}$. It follows that $u = 0$ on $\{z_n = 0\}$ and

$$F_\varepsilon(z) := f(z') \chi(|z_n|^2/\varepsilon^2) - u_\varepsilon(z)$$

is a holomorphic extension of f satisfying

$$\limsup_{\varepsilon \rightarrow 0} \int_{\Omega} \frac{|F_\varepsilon|^2 e^{-\varphi}}{|z_n|^2 \log^2 |z_n|^2} d\lambda \leq C \int_{\Omega'} |f|^2 e^{-\varphi} d\lambda'.$$

The required F is the weak accumulation point of F_ε . \square

4. The Suita conjecture and constants. Similarly as in [11], we consider a decreasing convex $\eta : \mathbb{R}_+ \rightarrow \mathbb{R}_-$ such that

$$(7) \quad \eta'' \geq \frac{(\eta')^2}{\eta + e^t}$$

and

$$(8) \quad C := \frac{1}{-\lim_{t \rightarrow \infty} \eta'(t)} < \infty.$$

An example of such an η is $-a(t + t^b)$, where $0 < b < 1$ and $a > 0$ is sufficiently small. The smallest C that can be obtained this way (numerically with *Mathematica*) is for η satisfying the equality in (7) and the initial condition $\eta(0) = 0$, $\eta'(0) = -2.216715\dots$; then $C = 1.95388\dots$

THEOREM 4. *Assume that $\Omega \subset \mathbb{C}^{n-1} \times D$ is pseudoconvex, where D is a bounded domain in \mathbb{C} containing the origin. Then for every plurisubharmonic φ in Ω and f holomorphic in $\Omega' := \Omega \cap \{z_n = 0\}$ (we assume that Ω' is not empty) there exists a holomorphic F in Ω such that $F|_{\Omega'} = f$ and*

$$\int_{\Omega} |F|^2 e^{-\varphi} d\lambda \leq \frac{C \pi}{(c_D(0))^2} \int_{\Omega'} |f|^2 e^{-\varphi} d\lambda',$$

where C is given by (8).

PROOF. First assume that $D = \{|\zeta| < 1\}$, so that in particular $c_D(0) = 1$. Let $0 < \varepsilon < 1$ and set

$$\alpha := \bar{\partial}(f(z')\chi(-2\log|z_n|)) = -f(z')\chi'(-2\log|z_n|)\frac{d\bar{z}_n}{\bar{z}_n},$$

where $\chi \in C^{0,1}(\mathbb{R}_+)$, such that $\chi(t) = 0$ for $t \leq M := -2\log \varepsilon$ and $\lim_{t \rightarrow \infty} \chi(t) = 1$, will be determined later. Further, set $\tilde{\varphi} := \varphi + 2\log|z_n|$ and $\psi := \gamma(-2\log|z_n|)$, where a convex decreasing $\gamma \in C^{1,1}(\mathbb{R}_+)$ will also be determined later. For u given by Theorem 1 we have

$$\int_{\Omega} |u|^2 e^{-\varphi} d\lambda \leq \int_{\Omega} (1 - |\bar{\partial}\psi|_{i\partial\bar{\partial}\psi}^2) |u|^2 e^{\psi - \tilde{\varphi}} d\lambda \leq \frac{1}{(1 - \sqrt{\delta})^2} \int_{\Omega} |\alpha|_{i\partial\bar{\partial}\psi}^2 e^{\psi - \tilde{\varphi}} d\lambda,$$

provided that

$$(9) \quad \left(1 - \frac{(\gamma')^2}{\gamma''}\right) e^{\gamma+t} \geq 1$$

on \mathbb{R}_+ (then the first inequality holds), $(\gamma')^2/\gamma'' \leq 1$ on \mathbb{R}_+ and $(\gamma')^2/\gamma'' \leq \delta < 1$ on $\{t \geq M\}$ (then the second inequality follows from Theorem 1).

Similarly as in the proof of Theorem 3, we have (recall that α , $\tilde{\varphi}$ and ψ depend on ε)

$$\limsup_{\varepsilon \rightarrow 0^+} \int_{\Omega} |\alpha|_{i\partial\bar{\partial}\psi}^2 e^{\psi - \tilde{\varphi}} d\lambda \leq \limsup_{\varepsilon \rightarrow 0^+} \mathcal{A}(\varepsilon) \int_{\Omega'} |f|^2 e^{-\varphi} d\lambda',$$

where

$$\mathcal{A}(\varepsilon) = \int_{\{|\zeta| \leq \varepsilon\}} \frac{(\chi'(-2\log|\zeta|))^2 e^{\gamma(-2\log|\zeta|)}}{|\zeta|^2 \gamma''(-2\log|\zeta|)} d\lambda(\zeta) = \pi \int_M^\infty \frac{(\chi')^2 e^\gamma}{\gamma''} dt.$$

The optimal choice of χ with $\int_M^\infty \chi'(t) dt = 1$ is

$$\chi(t) := \begin{cases} 0, & t \leq M \\ \frac{1}{c} \int_M^t w(s) ds, & t > M, \end{cases}$$

where $w = \gamma'' e^{-\gamma}$ and $c = \int_M^\infty w(s) ds$. Then

$$\mathcal{A}(\varepsilon) = \frac{\pi}{\int_M^\infty \gamma'' e^{-\gamma} dt}.$$

We now set

$$\gamma(t) := \begin{cases} -\log(-\eta(t)), & t \leq M \\ -\delta \log(t - M + a) + b, & t > M, \end{cases}$$

where a, b are chosen in such a way that $\gamma \in C^{1,1}(\mathbb{R}_+)$, that is

$$\begin{aligned} a &= a(M) = \delta \frac{\eta(M)}{\eta'(M)}, \\ b &= b(M) = -\log(-\eta(M)) + \delta \log a. \end{aligned}$$

We set $\delta = \delta(M) := M^{-1/2}$, so that in particular

$$(10) \quad \lim_{M \rightarrow \infty} a(M) = \infty.$$

One can easily check that on $\{t \leq M\}$ by (7) γ satisfies (9) and $(\gamma')^2/\gamma'' \leq 1$. On $\{t > M\}$ we have $(\gamma')^2/\gamma'' = \delta$ and for sufficiently large M , since $-\delta \log(t - M + a) + t$ is increasing in t ,

$$\left(1 - \frac{(\gamma')^2}{\gamma''}\right) e^{\gamma+t} \geq (1 - \delta) e^{-\log(-\eta(M)) + M} \geq 1.$$

Moreover,

$$\int_M^\infty \gamma'' e^{-\gamma} dt = \delta e^{-b} \int_0^\infty (t + a)^{\delta-2} dt = \frac{-\eta'(M)}{1 - \delta}$$

and this tends to $1/C$ as $M \rightarrow \infty$. Finally, we note that, if $0 < \tilde{\varepsilon} \leq \varepsilon$,

$$\begin{aligned} \int_{\{|\zeta| \leq \tilde{\varepsilon}\}} (1 - |\bar{\partial}\psi|_{i\bar{\partial}\bar{\partial}\psi}^2) e^{\psi - 2\log|\zeta|} d\lambda &= \pi \int_{-2\log\tilde{\varepsilon}}^\infty \left(1 - \frac{(\gamma')^2}{\gamma''}\right) e^\gamma dt \\ &= \pi(1 - \delta) e^b \int_{-2\log\tilde{\varepsilon} - M}^\infty (t + a)^{-\delta} dt \\ &= \infty \end{aligned}$$

which ensures that $u = 0$ on $\{z_n = 0\}$. Defining the extension F as in the proof of Theorem 3 gives the required result when $D = \{|\zeta| < 1\}$.

If D is arbitrary we set $G := G_D(\cdot, 0)$, α is defined as before, and we modify the definitions of $\tilde{\varphi}, \psi$ to $\tilde{\varphi} := \varphi + 2G, \psi := \gamma(-2G)$. We have to show that

$$(11) \quad \limsup_{\varepsilon \rightarrow 0^+} \int_{\{|\zeta| \leq \varepsilon\}} \frac{(\chi'(-2\log|\zeta|))^2 e^{\gamma(-2G) - 2G}}{4|\zeta|^2 |G_{\bar{\zeta}}|^2 \gamma''(-2G)} d\lambda(\zeta) \leq \frac{C\pi}{(c_D(0))^2}.$$

We see that

$$(12) \quad \lim_{\varepsilon \rightarrow 0^+} \sup_{\{|\zeta| \leq \varepsilon\}} |\zeta|^2 e^{-2G} = \frac{1}{(c_D(0))^2}.$$

We can write $G = \log|\zeta| + h$, where h is a harmonic function. Then for some constant A (independent of ε)

$$(13) \quad 4|\zeta|^2 |G_{\bar{\zeta}}|^2 = |1 + 2\bar{\zeta}h_{\bar{\zeta}}|^2 \geq (1 - A\varepsilon)^2$$

if ε is sufficiently small.

With the notation $t = -2 \log |\zeta|$, we have $|2G + t| \leq B$ for some constant B . We now modify the definition of M to $M := -2 \log \varepsilon - B$ and define γ as before. Then for $t \geq -2 \log \varepsilon$ we have $-2G \geq t - B \geq M$ and

$$(14) \quad \gamma(-2G) \leq \gamma(t - B) \leq \gamma(t) + \delta \log \frac{a + B}{a}$$

$$(15) \quad \gamma''(-2G) \geq \gamma''(t + B) \geq \left(\frac{a - B}{a} \right)^2 \gamma''(t).$$

We also have

$$(16) \quad \int_{M+B}^{\infty} \gamma'' e^{-\gamma} dt = \frac{-\eta'(M)}{1 - \delta} \left(\frac{a + B}{a} \right)^{\delta} \frac{a}{a - B/\eta'(M)}.$$

Combining (12)–(16) with (10) we now get (11). \square

Acknowledgements. Part of this research was done during author's visits to Tambara Institute of Mathematics of the University of Tokyo in October 2011 and to University of Tel Aviv in December 2011. Both were much needed breaks in his currently overwhelming administrative duties. He is grateful to Kengo Hirachi, Takeo Ohsawa and Semyon Alesker for these invitations.

Remark. After this paper was completed the optimal constant in the Suita conjecture and the Ohsawa–Takegoshi extension theorem was finally obtained in [7] building up on the methods developed here. However, Chen's proof from [8] of the extension theorem without optimal constant, presented here in a slightly different form, is probably the simplest one so far.

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Received December 20, 2012

Institute of Mathematics
 Jagiellonian University
 Łojasiewicza 6
 30-348 Kraków, Poland
e-mail: Zbigniew.Blocki@im.uj.edu.pl
 umblocki@cyf-kr.edu.pl